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B-Chromatic Number Of Line Graph Of Wheel Graph

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Abstract— A proper k-colouring of a graph G = (V (G), E (G)) is a mapping f: V (G) → N such that every two adjacent vertices receive different colours. The chromatic number of a graph G is denoted by χ(G), is the minimum number for which G has a proper k-colouring. The set of vertices with a specific colour is called a colour class. A b-colouring of a graph G is a variant of proper k-colouring such that every colour class has a vertex which is adjacent to at least one vertex in every other colour classes and such a vertex is called a colour dominating vertex. The b-chromatic number ϕ(G) is the largest integer k such that G admits a b-colouring with k colours. Here we investigated the b-chromatic number of Line graph of wheel graph.

Index Terms— Key Words: b-colouring,b-chromatic number, chromatic number,Line graph,proper coloring,wheel graph.

Notations: ϕ - b-chromatic number, χ- chromatic number

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1 Introduction

Many interesting problems have been determined by Graph colouring in graph theory The Four-Color Conjecture have been originated by Francis Guthrie. Let G be a simple graph with vertex set V (G) and edge set E (G). A colouring of the vertices of G is a mapping f: V (G) → N where N=1, 2,……k. For every vertex v ∈ V (G), f (v) is called the colour of v. If any two adjacent vertices have different colours then f is called proper colouring. The chromatic number χ(G) is the smallest integer k such that G admits a proper colouring using k colours. The set of vertices with a particular colour is called a colour class. A b-colouring by k colours is a proper colouring of the vertices of G such that in each colour class thereexists a vertex that has neighbours in all the other k-1 colour classes. In other words each colour class contains a vertex which has at least one neighbour in all the other colour classes. Such vertex is called colour dominating vertex. If v is a colour dominating vertex of a colour class c then we write cdv(c) = v. The b-chromatic number $\emptyset$(G) is the largest integer k such that G admits a b-colouring with k colours.

In 1999, Irving and Manlove [4] gave an introduction about the concept of colouring and showed that the problem of determining ϕ(G) is NP-hard for general graphs but it is polynomial-time solvable for trees. The bounds for the b-chromatic number for various graphs are endorsed by Kouider and Zaker in [6].The b-colouring of central graph of some graphs is analysed in Thilagavathi et al. In accordance with Faik the graph is b-continuous if b-colouring exists for every integer k satisfying $\chi(G) \leq k \leq \phi(G)$.. The b-chromatic number of Cartesian product of some families of graph is examined by Balakrishnan et al.[9] and b-colouring for square of Cartesian product of two cycles is discovered by Chandra Kumar and Nicholas .Some results on b-colouring and b-continuity are tabulated in Alkhateeb [8]. The b-chromatic number of some cycle related graph has examined by Vaidhya and Shukla and also they established the b-chromatic number of wheel related graph[11].The discussion about b-colouring was carried out by Amine El sahili and Mekkia kouider and they studied the b-chromatic number of a d-regular graph of girth 5.

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PRELIMINARIES:

Definition.2.1 [10]

A line graph L(G) of a simple graph G is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge iff the corresponding edges of G have a common vertex.

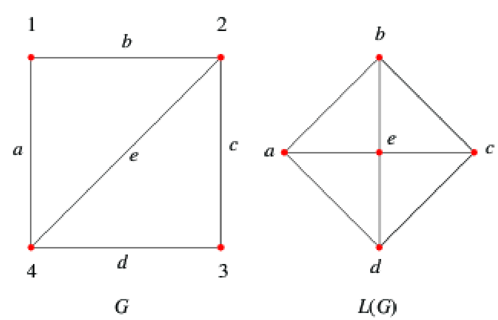

Fig.1.a fig.1.b

Fig.1.Line graph of a simple graph

***Definition* 2.2** [12]

Wheel Graph:

A wheel graph W_n is a graph with n vertices (n ≥ 4), formed by connecting a single vertex to all vertices of an (n-1)-cycle

Definition 2.3 [13]

Complete graph:

A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

Definition 2.4 [14]

Line graph of a wheel graph:

The following figure 2 illustrates a wheel graph and Line graph of wheel graph.

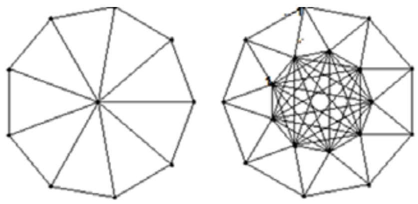

Fig.2.a fig.2.b

Fig 2: The wheel graph W_{10} and its line graph $L(W_{10})$

Proposition 2.1. [9] For any graph G, $\phi(G) \leq \Delta(G)+1$, where $\Delta(G)$ is the maximum degree of the graph G.

Proposition 2.2. [4] If a graph G admits a b-colouring with m-colours then G must have at least m vertices with degree at least m - 1 (Since each colour class has a b-vertex).

Lemma 2.3 If Kn be a complete graph, then $\phi(K_n) = n$, for all n.

Proof: Let K_n be a complete graph with n vertices. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ be vertex set of the complete graph K_n.Then $|V(G)| = n$ and $|E(G)| = n(n-1)/2$. To determined proper colouring we consider the following cases

Case 1: when n=3, $V(K_3)=\{v_1, v_2, v_3\}$, $|V(G)| = 3$, $|E(G)| = 3$

In this case, G has three vertices of degree 2.Maximum degree is 2.

Using Preposition 2.1, $\phi(K_3) \leq 3$

If $\phi(K_3) = 3$, as determined by preposition 2.2 the graph G must have three vertices of degree 2 which is possible.

To assign proper colouring and for b-colouring consider the colour set C={1,2,3} and define the colour function f:V⟶C such that $f(V_1) = 1, f(V_2) = 2, f(V_2) = 3$.

The above proper colouring enables cdv (1) = v_1; cdv (2) =V_2; cdv (3) = V_3; cdv (4) = v_4.

Therefore $\phi(K_3) = 3$.

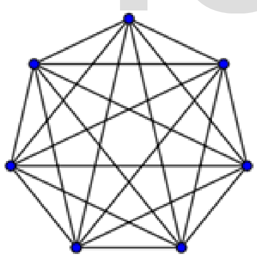

Fig.3

Case 2: when n=4, $V(K_4) = \{v_1, v_2, v_3, v_4\}$, $|V(G)| = 4$, $|E(G)| = 6$

In this case, G has four vertices of degree three. Maximum degree is 3 .

Using Preposition 2.1, $\phi(K_4) \leq 4$

If $\phi(K_4) = 4$,as determined by preposition 2.2 the graph G must have three vertices of degree 3 which is possible. To assign proper colouring and for b-colouring consider the colour set C={1,2,3,4} and define the colour function f:V→C such that $f(V_1)=1$, $f(V_2)=2$, $f(V_3)=3$, $f(V_4)=4$.

The above proper colouring enables cdv (1) = V_1 cdv (2) =V_2; cdv (3) = V_3; cdv (4) = V_4.

Hence $\phi(K_4) = 4$

From the above cases we concluded that the b-chromatic number of the complete graph with n vertices is n.

Hence $\phi(Kn) = n$.

3. MAIN RESULT:

Theorem: 3.1

If L (Wn) be the line graph of the wheel graph, then $\phi(L(W_n)) = n-1$.

Proof:

Let W_n be a wheel graph with n vertices with vertex set $V(w_n) = \{w_1, w_2, \ldots\ldots w_n\}$ with w_n as the hub. Now $L(W_n)$ contains a complete graph K_{n-1} with

$V(K_{n-1}) = \{u_1, u_2, \ldots\ldots u_{n-1}\}$ and a cycle C with $V(C) = \{v_1, v_2, \ldots\ldots v_{n-1}\}$.

To assign proper colouring and for b-colouring consider the colour set C= {1, 2,….n} and define the colour function f: V→C. Assign the colour ci to the vertex set $V(k_{n-1}) = u_i$ for i =1, 2 …n-1.

Next we have to colour the vertices of the cycle. If one more colour is introduced, say c_n it should be coloured to any one of the vertex of the cycle C. In the outer cycle C,v_1 is. adjacent with v_2, v_{n-1}, u_1, u_2. v_{n-1} is adjacent with $v_1, v_{n-2}, u_{n-1}, u_1$. In general each vertex v_i is adjacent with v_{i+1}, v_{i-1} for i=2,3…..n-1 and u_i, u_{i+1} for i=1,2,3…..n-1. Every vertex in the cycle has degree four. When the colour c_n is assigned to any vertex in the cycle which cannot harmonises its colour as c_n.Hence we can assign the n-1 colours which are assigned to the complete graph.

Next the vertices in the cycle v_i for i=3,4…..n-1 should assigned by the colour c_i for i=1, 2,……..n-3 and to the vertex v_1 should assigned by the colour c_{n-2} and v_2 by c_{n-1}. So, the new colours cannot be introduced.

The above proper colouring enables cdv (1) = u_1; cdv (2) =u_2; cdv (3) = u_3; cdv (n-1) = u_{n-1}.

Hence $\varphi(L(W_n)) = n$

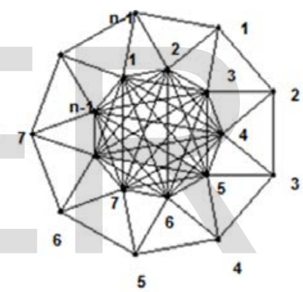

Fig. 4

4. CONCLUSION: Graph theory has wild applications in biochemistry, electrical engineering, computerscience and operations research. Graph colouring has many practical applications in scheduling, bandwidth allocation and pattern matching.Here we have obtained the b chromatic number of line graph of wheel graph.

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